The Cosmological Dynamics of Interacting Holographic Dark Energy Model

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Abstract

Motivated by the recent observations for the cosmic acceleration and the suitable evolution of the Universe provided an interaction (decay of dark energy to matter) is incorporated in a cosmological model, we study the cosmological evolution of the Interacting Holographic Dark Energy scenario. Critical points are derived and their corresponding cosmological models are presented. The dynamical character of these models is revealed.

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1 Introduction

Recent observations from type Ia supernovae [1] associated with Large Scale Structure [2] and Cosmic Microwave Background anisotropies [3] have provided main evidence for the cosmic acceleration. The combined analysis of cosmological observations suggests that the universe consists of about 70% dark energy, 30% dust matter (cold dark matter plus baryons), and negligible radiation. Although the nature and origin of dark energy are unknown, we still can propose some candidates to describe it, namely since we do not know where this dark energy comes from, and how to compute it from the first principles, we search for phenomenological models. Hopefully, astronomical observations will select one of them. The most obvious theoretical candidate of dark energy is the cosmological constant λ (or vacuum energy) [4, 5] which has the equation of state $w = -1$. However, as is well known, there are two difficulties that arise from the cosmological constant scenario, namely the two famous cosmological constant problems — the "fine-tuning" problem and the "cosmic coincidence" problem [6]. An alternative proposal for dark energy is the dynamical dark energy scenario. The dynamical dark energy proposal is often realized by some scalar field mechanism which suggests that the energy form with negative pressure is provided by a scalar field evolving down a proper potential. So far, a large class of scalar-field dark energy models have been studied, including quintessence [7], K-essence [8], tachyon [9], phantom [10], ghost condensate [11] and quintom [12], and so forth. But we should note that the mainstream viewpoint regards the scalar field dark energy models as an effective description of an underlying theory of dark energy. In addition, other proposals on dark energy include interacting dark energy models [13], braneworld models [14], Chaplygin gas models [15], and many others.

Currently, an interesting attempt for probing the nature of dark energy within the framework of quantum gravity (and thus compute it from first principles) is the so-called "Holographic Dark Energy" (HDE) proposal [16, 17, 18, 19]. It is well known that the holographic principle is an important result of the recent researches for exploring the quantum gravity (or string theory) [20]. The Holographic Dark Energy model has been tested and constrained by various astronomical observations [21, 22, 23, 24] as well as by the Anthropic Principle [25]. Furthermore, the Holographic Dark Energy model has been extended to include the spatial curvature contribution, i.e. the Holographic Dark Energy model in non-flat space [26]. For other extensive studies, see e.g. [27][1].

It is known that the coincidence or, "why now" problem is easily solved in some models of holographic dark energy based on this fundamental assumption that matter and holographic dark energy do not conserve separately, but the matter energy density decays into the holographic energy density [29, 30]. In fact a suitable evolution of the Universe is obtained when, in addition to the holographic dark energy, an interaction (decay of dark energy to matter) is assumed.

The remainder of the paper is organized as follows. In Section 2, we present the Holographic Dark Energy model where the interaction is included. Since the fractional contributions of the curvature (Ω_k), the ordinary matter (Ω_m), and the dark energy (Ω_X) do not form a compact state space, we introduce a new set of dimensionless variables which form such a compact state space. The evolution equations of these variables are derived and in order to study their dynamical character the critical points of this system are obtained.

[1] A very recent development is the idea of bulk holographic dark energy. In this proposal, holographic dark energy is accommodated in the framework of braneworld cosmology [28].

Cosmological models that correspond to the aforementioned critical points are presented and their dynamical character is given. At this point, it is worth noting that one of the properties that should characterize a cosmological solution is that the present universe is a global attractor, i.e. all the possible initial conditions lead to the observed percentages of dark energy and dark matter; once reached, they remain fixed forever [30]. This is the reason that here we will focus on the stability of the cosmological models that correspond to critical points and more importantly, these cosmological models are attractors. Finally, Section 3 is devoted to a brief summary of results and concluding remarks.

2 Stability of Interacting Holographic Dark Energy model solutions

In this section we consider the Holographic Dark Energy model when there is an interaction between the holographic energy density ρ_X and Cold Dark Matter (CDM) which has $w_m = 0$. Since we know neither the nature of dark energy nor the nature of dark matter, a microphysical interaction model is not available either. However, pressureless dark matter in interaction with holographic dark energy is more than just another model to describe an accelerated expansion of the universe. It provides a unifying view of different models which are viewed as different realizations of the Interacting Holographic Dark Energy Model at the perturbative level [31]. The continuity equations for dark energy and CDM are

$$\dot{\rho}_{\rm X} + 3H(1+w_{\rm X})\rho_{\rm X} = -Q, \tag{1}$$

$$\dot{\rho}_{\rm m} + 3H\rho_{\rm m} = Q \ . \tag{2}$$

The interaction is given by the quantity $Q = \Gamma\rho_X$ with Γ to have dimensions of pressure. This is a decaying of the holographic energy component into CDM with the decay rate Γ. Taking a ratio of two energy densities as $r = \rho_{\rm m}/\rho_{\rm X}$, the above equations lead to

$$\dot{r} = 3Hr\Big[w_{\rm X} + \frac{1+r}{r}\frac{\Gamma}{3H}\Big] \ . \tag{3}$$

It should be stressed that the decay rate has to be positive, i.e. $\Gamma > 0$, so as to interpret a transfer from the dark energy component to the matter component. Moreover, it is obvious from equation (3) that any restriction on the parameter of the equation of state for the holographic dark energy, i.e. $w_{\rm X}$, will set constraints on $\Gamma > 0$ [32].
Following [33], if we define

$$w_X^{\rm eff} = w_X + \frac{\Gamma}{3H} \ , \qquad w_m^{\rm eff} = -\frac{1}{r}\frac{\Gamma}{3H} \ , \tag{4}$$

then the continuity equations can be written in their standard form

$$\dot{\rho}_X + 3H(1+w_X^{\rm eff})\rho_\Lambda = 0 \ , \tag{5}$$

$$\dot{\rho}_m + 3H(1+w_m^{\rm eff})\rho_m = 0 \ . \tag{6}$$

We consider the non-flat Friedmann-Robertson-Walker universe with line element

$$ds^2 = -dt^2 + a^2(t)(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2) \tag{7}$$

where k denotes the curvature of space with k=0,1,-1 for flat, closed and open universe respectively. A closed universe with a small positive curvature ($\Omega_k \sim 0.01$) is compatible with observations [34, 35]. We employ the Friedmann equation to relate the curvature of the universe to the energy density. The first Friedmann equation is given by

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2}\Big[\rho_{\rm X} + \rho_{\rm m}\Big] \ . \tag{8}$$

We define the dimensionless fractional contributions of CDM, holographic dark energy, and curvature as follows

$$\Omega_{\rm m} = \frac{\rho_m}{\rho_{cr}} = \frac{\rho_{\rm m}}{3M_p^2H^2} \ , \qquad \Omega_{\rm X} = \frac{\rho_X}{\rho_{cr}} = \frac{\rho_{\rm X}}{3M_p^2H^2} \ , \qquad \Omega_k = \frac{k}{a^2H^2} \ . \tag{9}$$

Using the results of [36], the effective parameters of the equation of states given in equation (4) are now expressed in terms of the above-mentioned dimensionless parameters as

$$w_{\rm X}^{eff} = -\frac{1}{3} - \frac{2\sqrt{\Omega_{\rm X} - c^2\Omega_k}}{3c} \ , \tag{10}$$

$$w_m^{eff} = \frac{-b^2(1+\Omega_k)}{\Omega_{\rm m}} \ . \tag{11}$$

At this point it should be noted that for the computation of the effective parameters as given above, the decay rate is chosen to be

$$\Gamma = 3b^2\,(1+r)\,H \tag{12}$$

where b is a dimensionless coupling constant, and the holographic dark energy density is of the form

$$\rho_X = 3c^2M_p^2L^{-2} \tag{13}$$

where c is a positive constant which can not be less than 1. The IR cut-off parameter L is chosen to be the radius of the event horizon as measured on the sphere of the horizon and is defined as

$$L = ar(t) \tag{14}$$

where $r(t)$ is obtained from

$$\int_0^{r(t)} \frac{dr}{\sqrt{1-kr^2}} = \frac{R_h}{a} \tag{15}$$

which yields

$$r(t) = \frac{1}{\sqrt{k}}\sin y \tag{16}$$

with $y = \sqrt{k}R_h/a$ and R_h is the radius of the event horizon measured in the r direction

$$R_h = a\int_t^\infty \frac{dt}{a} \ . \tag{17}$$

Now we describe solutions of the model under investigation and also determine their stability. To do this, we use a set of convenient phase-space variables similar to those introduced in [37, 38]. We introduce the notation similar to those of [37], and work in

the 3-dimensional Ω-space $(\Omega'_k, \Omega_m, \Omega_{\rm X})$ where $\Omega'_k = -\Omega_k$. We can now rewrite the first Friedmann equation as

$$\Omega_{\rm m} + \Omega_\Lambda + \Omega'_k = 1 \ . \tag{18}$$

It is obvious that when the universe is flat ($k = 0$) or open ($k = -1$), all terms in equation (18) that are put together to give unity are nonnegative and take values in $[0, 1]$. Nonetheless all of them are singular in the case of $H = 0$. For the case of closed universe ($k = 1$) which is of our interest, the terms in equation (18) are not anymore all nonnegative. Therefore, the state space defined by the variables $(\Omega'_k, \Omega_m, \Omega_{\rm X})$ is no longer compact (because now $\Omega'_k < 0$ for $k = 1$). However, we can employ another set of variables describing a compact state space. Therefore, in lieu of using the Hubble function H, we introduce the following quantity

$$D = \sqrt{H^2 + \frac{k}{a^2}} \tag{19}$$

and thus we define the following dimensionless variables

$$Z = \frac{H}{D} \ , \tag{20}$$

$$\tilde{\Omega}_m = \frac{\rho_{\rm m}}{3M_p^2 D^2} \ , \tag{21}$$

$$\tilde{\Omega}_X = \frac{\rho_{\rm X}}{3M_p^2 D^2} \ . \tag{22}$$

From these definitions it is evident that now the case $H = 0$ is included. Moreover, the Friedmann equation takes the following form

$$\tilde{\Omega}_m + \tilde{\Omega}_X = 1 \tag{23}$$

which, together with the fact that $-1 \leq Z \leq 1$ since the Hubble parameter can be positive ($H > 0$) for an expanding cosmological model or negative ($H < 0$) for a contracting model, implies that the state space defined by the new variables is indeed compact. Introducing the primed time derivative

$$' = \frac{1}{D}\frac{d}{dt} \tag{24}$$

one obtains the system of first-order differential equations

$$D' = -Z^3 D\left(q + \frac{1}{Z^2}\right) \ , \tag{25}$$

$$Z' = Z^2\left[-1 - q + Z^2\left(q + \frac{1}{Z^2}\right)\right] \ , \tag{26}$$

$$\tilde{\Omega}'_m = \tilde{\Omega}_m Z\left[-3\left(1 + w_m^{\rm eff}\right) + 2Z^2\left(q + \frac{1}{Z^2}\right)\right] \ , \tag{27}$$

$$\tilde{\Omega}'_X = \tilde{\Omega}_X Z\left[-3\left(1 + w_X^{\rm eff}\right) + 2Z^2\left(q + \frac{1}{Z^2}\right)\right] \ , \tag{28}$$

where

$$q = \frac{-\ddot{a}}{H^2 a} = -\left(\frac{\dot{H}}{H^2} + 1\right) = \frac{3\left(\tilde{\Omega}_X w_X^{\rm eff} + \tilde{\Omega}_m w_m^{\rm eff}\right)}{2Z^2} + \frac{1}{2Z^2} \ . \tag{29}$$

The evolution equation for D is not coupled to the rest of the equations (26-28), so we will not consider it for our dynamical study. Therefore, we will only study the dynamical system for the variables $\tilde{\mathbf{\Omega}} \equiv (Z, \tilde{\Omega}_m, \tilde{\Omega}_X)$, determined by the equations (26-28). The behavior of this system of equations in the neighborhood of its stationary point is determined by the corresponding matrix of its linearization. The real parts of its eigenvalues tell us whether the corresponding cosmological solution is stable or unstable with respect to the homogeneous perturbations [39].
To begin with, we have to find the critical points of this dynamical system, which can be written in vector form as follows

$$\tilde{\mathbf{\Omega}}' = \mathbf{f}(\tilde{\mathbf{\Omega}}) \tag{30}$$

where f can be extracted from equations (26-28). The critical points, $\tilde{\mathbf{\Omega}}^*$, namely the points at which the system will stay provided it is initially at there, are given by the condition

$$f(\tilde{\mathbf{\Omega}}^*) = \mathbf{0} \ . \tag{31}$$

Their dynamical character is determined by the eigenvalues of the matrix

$$\left.\frac{\partial \mathbf{f}}{\partial \tilde{\mathbf{\Omega}}}\right|_{\tilde{\mathbf{\Omega}} = \tilde{\mathbf{\Omega}}^*} . \tag{32}$$

The coordinates of the critical points in the state space, i.e. $\tilde{\mathbf{\Omega}} = (Z, \tilde{\Omega}_m, \tilde{\Omega}_X)$, as well as their eigenvalues are given in the following table

Model	Coordinates	Eigenvalues
F_-	$(-1, 1, 0)$	$-(1 + 3w_m^{\rm eff}, 3w_m^{\rm eff}, 3(w_m^{\rm eff} - w_X^{\rm eff}))$
dS_-	$(-1, 0, 1)$	$-(1 + 3w_X^{\rm eff}, 3(w_X^{\rm eff} - w_m^{\rm eff}), 3w_X^{\rm eff})$
E	$(0, -\frac{(1+3w_X^{\rm eff})}{3(w_m^{\rm eff} - w_X^{\rm eff})}, \frac{(1+3w_m^{\rm eff})}{3(w_m^{\rm eff} - w_X^{\rm eff})})$	$\alpha(0, -1, 1)$
F_+	$(1, 1, 0)$	$(3w_m^{\rm eff}, 1 + 3w_m^{\rm eff}, 3(w_m^{\rm eff} - w_X^{\rm eff}))$
dS_+	$(1, 0, 1)$	$(3w_X^{\rm eff}, 3(w_X^{\rm eff} - w_m^{\rm eff}), 1 + 3w_X^{\rm eff})$

with

$$\alpha = \sqrt{\frac{-(1 + 3w_m^{\rm eff})(1 + 3w_X^{\rm eff})}{2}} \ . \tag{33}$$

The cosmological models identified with F_+ and F_- are the expanding ($H > 0$) and contracting ($H < 0$) FLRW models ($k = \Lambda = 0$) respectively, dS_+ and dS_- are the expanding and contracting de Sitter models ($k = \rho_m = 0$) respectively, and E stands for the Einstein universe ($k = 1$ and $H = 0$).
Concerning the dynamical character of the critical points, if the real part of the eigenvalues of a critical point is not zero, the point is said to be *hyperbolic* [37]. In this case, the dynamical character of the critical point is determined by the sign of the real part of the eigenvalues: if all of them are positive, the point is said to be a *repeller*, because arbitrarily small deviations from this point will move the system away from this state. If all of them are negative the point is called an *attractor* because if we move the system slightly from this point in an arbitrary way, it will return to it. Otherwise, we say the critical point is a *saddle* point. As we can see from the previous table the dynamical character of all equilibrium points depend on the equation of state parameters, i.e. $w_m^{\rm eff}$ and $w_X^{\rm eff}$.

Model	Repeller	Attractor
F_-	$w_m^{\rm eff} < -\frac{1}{3}$ and $w_m^{\rm eff} < w_X^{\rm eff}$	-
dS_-	$w_X^{\rm eff} < -\frac{1}{3}$ and $w_m^{\rm eff} > w_X^{\rm eff}$	-
E	-	-
F_+	-	$w_m^{\rm eff} < -\frac{1}{3}$ and $w_m^{\rm eff} < w_X^{\rm eff}$
dS_+	-	$w_X^{\rm eff} < -\frac{1}{3}$ and $w_m^{\rm eff} > w_X^{\rm eff}$

It should be pointed out that for the Einstein universe (E-model) there is no way to be a repeller or, an attractor. It represents a set of infinite saddle points which have to satisfy one of the following conditions

$$w_m^{\rm eff} > -\frac{1}{3} \quad \text{and} \quad w_X^{\rm eff} < -\frac{1}{3} \tag{34}$$

$$\text{or,} \quad w_m^{\rm eff} < -\frac{1}{3} \quad \text{and} \quad w_X^{\rm eff} > -\frac{1}{3}\ . \tag{35}$$

3 Conclusions

Understanding the dark energy is one of the biggest challenges to the particle physics this century. Studying the interaction between the dark energy and ordinary matter will open a possibility of detecting the dark energy. Being a dynamical component, the scalar field dark energy is expected to interact with the ordinary matters. For example, Carroll [40] (see also [41]) has considered an interaction of form $QF_{\mu\nu}\tilde{F^{\mu\nu}}$ with $F_{\mu\nu}$ being the electromagnetic field strength tensor which has interesting implication on the rotation of the plane of polarization of light coming from distant sources. Recent data on the possible variation of the electromagnetic fine structure constant has triggered interests in studies related to the interactions between quintessence and the matter fields.

In this paper we have studied the cosmological dynamics of the Interacting Holographic Dark Energy model. We derived the system of the first-order differential equations which describes the evolution of the three dimensionless quantities that form a compact state space. Furthermore, the five critical points of the aforementioned cosmological model were obtained and the dynamical character of these critical points was presented.

In particular, it was shown that E-model represents a set of infinite saddle points whose line element is that of the Einstein universe. By using the fact that the equation of state parameters w_m^{eff} and w_X^{eff} should always be negative as indicated by equations (10) and (11), we obtained that if $\left|3w_m^{eff}\right| < 1$ the critical points of the $F_\pm$ models are saddle points. On the other hand, if $\left|3w_m^{eff}\right| > 1$, the critical point of model F_- will be a repeller under the condition $|w_m^{eff}| > |w_X^{\rm eff}|$, while the critical point of the F_+ model will be an attractor. Concerning the $dS_\pm$ models, we see that if $|3w_X^{eff}| < 1$ then the critical points of $dS_\pm$ are saddle points. On the other hand, if $|3w_X^{eff}| > 1$, the critical point of model dS_- model will be a repeller under the condition $|w_m^{eff}| < |w_X^{\rm eff}|$, while the critical point of dS_+ will be an attractor.

Finally, it should be stressed that evidence was recently provided by the Abell Cluster A586 in support of the interaction between dark energy and dark matter [42]. However, despite the fact that numerous works have been performed till now, there are no strong observational bounds on the strength of this interaction [43]. This weakness to set stringent (observational or theoretical) constraints on the strength of the coupling between

dark energy and dark matter stems from our unawareness of the nature and origin of dark components of the Universe. It is therefore more than obvious that further work is needed to this direction.

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